INTRA-MODE COUPLING LENGTH ANALYSIS IN FEW-MODE FIBERS

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A method to assess the coupling length within groups of degenerate modes in few-mode fibers is proposed and applied to a case scenario where a step-index fiber is affected by stress birefringence and core ellipticity.

Keywords: few-mode fiber, mode-coupling

1. Introduction

In mode-division multiplexing systems, transmission is impaired by coupling occurring among modes and can only be accomplished in the two opposite regimes of weak- and strong-coupling, each requiring a different strategy at the receiver to recover the data [1]. A precise characterization of coupling is thus fundamental to design the system. This work proposes a method to assess coupling in terms of coupling length L_C, which is the distance at which the average power $p(z)$ launched on a mode decorrelates, spreading into the others. This parameter, when compared with the fiber length L, indicates the coupling regime: weak if $L_C \gg L$, or strong if $L_C \ll L$. Because of the complex behavior that power may assume when coupling occurs, we propose a definition of coupling length sensitive to possible power oscillations, i.e., $L_C = \int \tilde{p}(z)dz/\tilde{p}(0)$, with $\tilde{p}(z) = p(z) - p(\infty)$. Hence, L_C takes shorter values for oscillations of higher amplitudes and frequencies. Since coupling is much stronger within groups of degenerate modes, we neglect inter-group coupling to simplify the analysis. In a degenerate group, the propagation is given by $\mathrm{d}\boldsymbol{a}/\mathrm{d}z = -j\boldsymbol{R}(\theta)\boldsymbol{K}_0\boldsymbol{R}^T(\theta)\boldsymbol{a}$ [2], where $\boldsymbol{a}$ are the amplitudes of the modes, $\boldsymbol{K}_0$ is the coupling matrix when the perturbation is aligned to the reference frame of the fiber, $\boldsymbol{R}$ is a proper rotation matrix, and $\theta(z)$ is the angle at which the perturbation is tilted with respect to the reference. Following the *fixed-modulus model* [2], we choose $\theta(z)$ as a *Wiener process*, i.e., $\mathrm{d}\theta/\mathrm{d}z = -\sigma\eta(z)$ with $\eta(z)\sim\mathcal{N}(0,1)$ normal distributed. The perturbation's strength and orientation dynamics are outlined respectively by two parameters: the *coupling beat length*, $L_K = 2\pi/(\max\kappa_i - \min\kappa_i)$, with $\kappa_i \in \mathrm{eig}\{\boldsymbol{K}_0\}$; and the *fiber correlation length*, $L_F = 1/(2\sigma^2)$. Short values of L_K and L_F indicate strong and fast evolving perturbations, and conversely with long values the opposite is true.

2. Results

The model can be cast as a stochastic differential equation [3], where the state of the system $\boldsymbol{x}$ includes both $\boldsymbol{a}$ and θ. In this context, the average power for the i-th mode is $p_i(z) = \boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{y}(\boldsymbol{x})$, where a set of auxiliary function $\boldsymbol{y}$ are linearly combined according to $\boldsymbol{c}_i$. The Dynkin's formula [4] provides a set of differential equations for $\boldsymbol{y}$, $\mathrm{d}\boldsymbol{y}/\mathrm{d}z = \boldsymbol{M}\boldsymbol{y}$; we can then write $p_i(z) = \sum_j w_{ij}\exp(\lambda_j z)$, where λ_j are the eigenvalues of $\boldsymbol{M}$ and w_{ij} are linear combinations of the eigenvectors of $\boldsymbol{M}$, of $\boldsymbol{c}_i$, and of the launching conditions. Then, the coupling length L_C^i for the i-th mode becomes $L_C^i = (\sum_j w_{ij}/\lambda_j)/\sum_j w_{ij}$, where the sum excludes $\lambda_j = 0$ to not consider the asymptotic value. The method is used to analyze coupling within groups of degenerate LP modes in step-index fibers when affected by stress-birefringence and core ellipticity. By inspecting the coupling matrices $\boldsymbol{K}_0$, four different cases are identified [2]: 1) LP_{0p} groups with birefringence or ellipticity; 2) LP_{np} $n \neq 0$ groups with birefringence, and LP_{np} $n \geq 3$ groups with ellipticity; 3) LP_{1p} modes with ellipticity; 4) LP_{2p} modes with ellipticity. For each case, the coupling length of a mode when all the power is launched on that mode is evaluated at different values of L_F/L_K and represented, normalized over L_K, in Fig. 1. Results show two asymptotic regions, $L_F \ll L_K$ and $L_F \gg L_K$, where $L_C \approx qL_F^m/L_K^{m-1}$. An estimate of the parameters yields $m = -1$ and $0.05 \leq q \leq 2.3\cdot10^3$ in the first, and $m = 1$ and $0.08 \leq q \leq 0.25$ in the second one. Remarkably, modes in case 1) and 2) have the same coupling length due to the peculiar diagonal form of their coupling matrix. Finally, all the modes in the same group have equal L_C except for the LP_{2p} ones when the fiber slightly elliptical. In this case (4), the x-polarized modes behave differently from the y-polarized ones. These results provide a deeper understanding of the coupling mechanisms and are useful to design the transmission system.

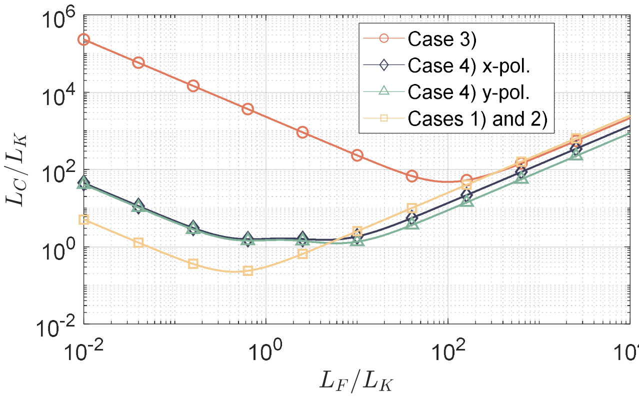

Fig. 1 Coupling length L_C normalized over L_K as a function of L_F/L_K.

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References

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